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Problem 1

The Quantified Boolean Formula consists of logical connectives (\vee , \wedge , \neg , \Rightarrow), boolean constants \top , \perp , variables and quantifiers. All are boolean, so $\forall_x \phi$ means *the formula ϕ holds for every logical value substituted for x* . An example true QBF formula is

$$(\forall_x x \vee \neg x) \wedge (\forall_x \exists_y x \Rightarrow y).$$

We say that QBF is well formed if every variable is in the scope of some quantifier. We consider only well formed QBFs. All transformations of QBFs in this task should work in polynomial time. Here are some facts about QBF formulas:

- F1 every QBF ϕ can be transformed to the equivalent QBF ϕ' in which all variables in the quantifiers are unique,
- F2 every QBF ϕ can be transformed to the equivalent QBF ϕ' , such that $\phi' = (Q_1 x_1 Q_2 x_2 \dots Q_n x_n \psi)$, where $Q_i \in \{\forall, \exists\}$, and ψ is a standard logical formula (without quantifiers).

Solve the following assignments:

- I Prove fact F1
- II Prove fact F2. You can refer to F1.
- III Show how to use MiniMax algorithm to check whether a QBF is true. You should describe the moves in the game (for Min and Max), tell what is a game state, which player starts, tell when the game finishes and what is the value of the terminal state. You can use F1 and F2.

Problem 2

Let us consider the rectangular grid world (of size $M \times M$) in which agent can move if 4 directions (N, E, S, W). Agent has stamina points (integer number), every move decreases their stamina by one. Agent with zero stamina cannot move, but there are some places (chests with food) c_1, \dots, c_n which increases stamina by values v_1, \dots, v_n (when the agent goes on them). Every such place works only once. Moreover, K squares contain obstacles, agent cannot move on them. The goal of the agent is to reach special target square (and find a treasure) with maximal possible stamina.

1. This task can be described as a search problem in a state space. How many states we need?
2. What is the A^* algorithm (give one sentence description)? Give two heuristic functions h_1 and h_2 which can be used in A^* in this task. Both these functions have to be connected with some relaxations of the task. Clarify this issue.
3. Let us suppose that the values of food chests are random (and agents knows the distribution for every chest). Describe the agent who use Monte Carlo simulations and can change their plan after gathering from the environment information about the drawn value (for particular food chest).