University of Wrocław: Data Science 15.09.2020

Entrance exam: Introduction to Computer Science

First name: .... Last name: ....

General Remark. The scores for algorithms designed as solutions for particular problems will depend on correctness as well as their efficiency, especially time complexity.

1. (**7 pts**) Consider the following problem. Given a sequence of (possibly negative) integers $a_1, \ldots, a_n$ determine the largest value of the sum of consecutive elements, i.e., $\max_{1 \le i \le j \le n} \{\sum_{k=i}^{j} a_k\}$.

   Design an algorithm which solves the problem, prove correctness of your algorithm, determine its asymptotic time and space complexity.

2. (**6 pts**) Assume that a set of natural numbers $X = \{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \cdots < x_n$ is stored in a binary search tree T. That is, for a node v of T with the value x of its key, the left subtree of v contains only nodes with values smaller than x and the right subtree contains only nodes with values larger than x.

   Design an algorithm which, for a given input number s, returns the closest to s element of X. That is, your algorithm should outup such number $x_j \in X$ that $|x_j - s| = \min_{i=1,2,\ldots,n}(|x_i - s|)$.

   Give a pseudocode of your algorithm, describe its idea and evaluate its time complexity.

   Remark. Time complexity of your algorithm can be expressed as a function of n and h, where h is the height of T.

3. (**7 pts**) Consider the following algorithm Surprise:

Algorithm 1 Surprise(n, m)

```
if n = 0 then
    return 0
r ← n mod m
p ← n div m
if r > 0 then
    return 0
else
    return 1 + Surprise(p, m)
```

where a mod b is the remainder of the integer division a/b and a div b is the result of the integer division a/b. (For example, $7 \bmod 2 = 1$ and $7 \operatorname{div} 2 = 3$.)

(a) (1 pt) Put the values returned by Surprise(n, m) in the table for the given values n and m:

n (rows) / m (col.)	2	3	4
8			
12			
$2^{11} \cdot 3^{17}$			

(b) (2 pts) Let $T(n, m)$ be the asymptotic worst-case time complexity of the function Surprise. Fill the gaps in the recurrence relation defining T for natural $n \geq 0$ and $m > 1$.

$$\begin{aligned} T(0, m) &= 1 \\ T(n, m) &= T(\ldots\ldots\ldots\ldots\ldots) + \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{for } n \geq 1 \end{aligned}$$

You can use mod and div operators in your solution.

(c) (2 pts) Give explicitely the asymptotic worst-case time complexity $T(n, m)$ of Surprise. Prove correctness of your answer.

Hint. You can e.g. resolve the recurrence relation from (b) to get the answer.

(d) (2 pts) Fill the gaps to complete the specification of the algorithm Surprise.

Input: n – a nongative integer
m – a positive integer

Output: ....

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