University of Wrocław: Data Science	15.09.2020

Entrance exam: STATISTICS AND LINEAR MODELS

First name: .... Last name: ....

1. (**6 pts**)

   Let $X_1, \ldots, X_n$ be i.i.d. random variables from the distribution with the density $f(x;\alpha) = (\alpha+1)x^{\alpha}$, for $x \in (0,1)$, $\alpha > 0$. Find the estimating equation for the maximum likelihood estimator $\hat{\alpha}_n$ of the parameter α.

2. (**7 pts**)

   Let $X_1, \ldots, X_n$ be i.i.d. random variables from the exponential distribution with the density $f(x;\theta) = \lambda e^{-\lambda x}$, for $x > 0$, $\lambda > 0$. Find the uniformly most powerful test at the level $\alpha = 0.05$ for testing the hypothesis $H_0 : \lambda = 3$ against $H_1 : \lambda = 5$. What is the power of the test under the alternative?

3. (**7 pts**)

   Consider the data set with $n = 20$ observations and two variables: Y, X. Assume that the relation between values of the response variable Y and the explanatory variable X is provided by the following model

   $$\text{for } \ i \ \in \{1, \ldots, n\} \ \ Y_i = \beta X_i + \epsilon_i \ \ , \tag{1}$$

   where $\epsilon_1, \ldots, \epsilon_n$ are iid random variables from the standard normal distribution $N(0, \sigma^2)$.

   a) Derive (5pt) (or provide, 2pt) the formulas for the maximum likelihood estimators $\hat{\beta}$ and $\hat{\sigma}^2$ of β and σ.

   b) Derive (2pt) (or provide, 1pt) the formula for the distribution of $\hat{\beta}$.